Discrete moduli for type I compactifications

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(Received 29 June 2001; published 26 December 2001)

We study type I compactification on a 4-torus, with a nontrivial discrete background RR 4-form field. By using string dualities and recent insights for gauge theories on tori, we find that a nontrivial background for the RR 4-form is correlated with $Spin(32)/\mathbb{Z}_2$ bundles that are described by a "nontrivial quadruple" of holonomies. We also briefly discuss other discrete moduli for the type I string, and variants of orientifold planes.

DOI: 10.1103/PhysRevD.65.026007 PACS number(s): 11.25.Mj

I. INTRODUCTION

The type I string can be regarded as an orientifold of the type IIB string. In this construction one introduces an $O9^-$ orientifold plane in the theory. This however causes a tadpole, and consistency requires that this is canceled by adding 32 D9 branes. The resulting theory has unoriented closed strings, while the D9-branes introduce an open string sector, leading to a theory with a gauge group that has as its manifest gauge group $O(32)$. Also a number of solitons survive the orientifold projection, leading to a spectrum with D1, D5 and D9 branes.

Interestingly, some unstable D-brane–antibrane configurations of type IIB theory become stable under the orientifold projection: The tachyon arising from the string states connecting branes and antibranes is projected out [1]. Therefore, new non-Bogomol'nyi-Prasad-Sommerfield (BPS) branes enter the theory. These nonperturbative objects introduce states in the theory that transform in different representations of the gauge group than the states arising from the ordinary open string sector. It is argued that these fix the topology of the gauge group to $Spin(32)/\mathbb{Z}_2$, the gauge group of one of the heterotic theories [2]. Indeed, the existence of these nonperturbative states is crucial evidence for the conjecture that the type I and the $Spin(32)/\mathbb{Z}_2$ heterotic string are S dual, and therefore in reality describe two limits of the same theory [3].

In type IIB supergravity, the low energy theory to the type IIB superstring, one encounters a variety of tensorfields, which couple to the extended objects in the theory [4]. Introducing the orientifold plane, many of these fields can no longer fluctuate because that would be incompatible with the orientifold projection. The fluctuating fields that survive the orientifold projection are in one-to-one correspondence with the BPS D-branes that appear in type I theory. The fields whose fluctuations are projected out are constrained to take constant values over all of space-time. That constant value does however not necessarily have to be equal to zero. It is argued that, before the orientifold projection, these fields are $U(1)$ valued; the orientifold projection acts as inversion on the circle which is the group manifold, and therefore has two fixed points. One can therefore argue that $U(1)$ is broken to $\mathbb{Z}_2$. The fact that $\mathbb{Z}_2$ is a discrete group demonstrates that continuous fluctuations are no longer possible, but it leaves a possibility for non-trivial values for these background fields.

The relevant tensorfields are gauge fields, and the field strengths corresponding to them must vanish because of the constancy of the potentials. It is nevertheless possible to construct gauge invariant operators, that can have nontrivial values if space-time has compact submanifolds. Indeed, the tensor fields are n-forms, and integrating these over compact n-cycles gives us gauge invariant operators. For a 1-form such an operator would correspond to the standard definition of holonomy. For an n-form one has $(n-1)$ gerbe-holonomy.

A case which has by now been well studied is the possibility for the Neveu-Schwarz–Neveu-Schwarz (NS-NS) 2-form field B_2 to have a nontrivial value over compact 2-cycles [5]. This gives a nontrivial phase to closed string world sheets that wrap around the relevant 2-cycle. Alternatively, this closed string world sheet can be interpreted as a collection of open string world sheets (disks), with the boundaries glued together. Reproducing the phase factor then places restrictions on the Chan-Paton bundle, and in fact one can show that the $Spin(32)/\mathbb{Z}_2$ bundle should be topologically nontrivial [6] (even when the 2-cycle is a 2-torus, and the bundle is flat [7]). Such bundles have "absence of vector structure" because states transforming in the vector representation of $Spin(32)$ cannot be consistently introduced in such a background. Of course, such states are argued to be absent in type I string theory, and the resulting compactification is consistent.

In the case that the 2-cycle is (topologically) a 2-torus it is possible to study the bundle [8], and consequently the type I string theory with such a bundle [7] in great detail (see also [9]). In contrast, there is little known about string compactifications with nontrivial backgrounds from the other discrete moduli.

In the present paper we will consider the possibility of a nontrivial background from the Ramond-Ramond (RR) 4-form. Unlike B_2, this does not appear in perturbative string theory, but it should generate nonperturbative effects. It would be very interesting to describe compactification on general 4-cycles allowing one to turn on this background, but we do not know how to do this at present. For the special case of 4-tori, there are however some recent developments, that make a study accessible. New insights in constructing

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flat bundles on a 3-torus have revealed that the moduli space of flat connections is much richer than previously thought [10–12]. Although this research has not (yet) been extended to cover 4- and higher dimensional tori, there are some partial results [11,13,14] that throw sufficient light on the theory we are interested in, the $Spin(32)/\mathbb{Z}_2$ gauge theory appearing in the type I string. Armed with these, and string dualities we will describe type I compactifications on a 4-torus, with nontrivial background from the RR 4-form. On the 4-torus one can of course also turn on a nontrivial NS-NS 2-form, and we will describe also these compactifications.

II. D-BRANES IN BACKGROUND NS-NS AND RR FIELDS

We will briefly review some aspects of toroidal compactification with nontrivial holonomy for the NS-NS 2-form, and point out some parallels and differences with the case we are interested in, holonomy for the RR 4-form.

For the NS 2-form, it is convenient to first study its effect in generality, and only afterwards introduce orientifold planes. A very simple way to study what happens when one turns on a nonzero B_2-field over some 2-cycles of an n-torus is by using T duality. Under T duality the metric and B_2-field moduli mix (see [15] for a review), and the dual torus has angles different from the original one. Consider a square 2-torus, with an appropriately normalized B_2-field. Let there also be a D-brane wrapping the 2-torus (possibly multiple times) and suppose there is a field strength F_2 present on the brane. Set

$$\int_{T^2} B_2 = b; \quad \int_{T^2} F_2 = f.$$

Let there be a single D-brane wrapping the 2-torus (possibly multiple times). Applying a single T duality, the dual torus becomes a skew one. The angle ϕ between two basis vectors for the lattice for this torus is given by $\tan\phi = b$. The single D-brane is dualized to a brane wrapping one cycle of the torus. This brane makes an angle ψ, given by $\tan\psi = -f$ with the other cycle. Closure of the D-brane now translates into the condition

$$n(b+f) \in \mathbb{Z} \quad \text{for} \quad n \in \mathbb{Z}. \tag{1}$$

The number n is appropriately interpreted as "wrapping number" of the D-brane. This obviously implies that $b+f$ is a rational number. Solutions with $f=0$ exist if and only if b is a rational number. Assume this to be the case and let m be the smallest integer such that $mb \in \mathbb{Z}$. Then a D-brane wrapping the dual torus m-times, with $\psi = 0$ describes the dual theory to a D-brane wrapping the original torus multiple times, which has on its world volume a gauge theory which is described by an $U(m)$ bundle with twisted boundary conditions. These boundary conditions break the $U(m)$ to $U(1)$, justifying in hindsight our ignoring of the non-Abelian interactions. As a side remark, we note that in the general case, where we wrap the brane multiple times and introduce a nonzero field strength, one can proceed by decomposing $U(m)$ as $[U(1) \times SU(m)]/\mathbb{Z}_m$. One then uses twisted boundary conditions in $SU(m)$, and puts the field strength in the $U(1)$ factor. Again, this breaks the effective gauge group to $U(1)$, and our ignoring of the non-Abelian interactions is justified.

By T duality the conclusions from the previous paragraph can be translated back to the original theory. When one wraps a brane over a 2-torus with nontrivial B_2-holonomy one should compensate for the effects by turning on a field strength on the brane, and/or wrapping the brane multiple times. The same conclusion can be reached from a more advanced argument. Consider a string world sheet ending on the D-brane, wrapped around a 2-cycle. In the path integral there appears a phase factor [17]

$$\exp\left(i \int_{\Sigma} B + i \oint_{\partial\Sigma} A \right). \tag{2}$$

In [17] an additional factor was considered, coming from the Pfaffian of the Dirac operator on the world sheet, but this is not relevant to our present considerations. For reasons explained below, the total factor should be equal to unity. Of course $\oint_{\partial\Sigma} A$ can simply be converted into $\int_{\Sigma} F_2$. Then, trivializing the phase factor requires turning on an appropriate field strength, and/or wrapping the brane multiple times over the cycle. This extends the previous result to generic 2-cycles.

In a somewhat more specific context, a similar argument was already used in [6]. Here type I theory on a K3 was considered, where the 2-cycles have the topology of spheres. Type I theory only allows B_2 fields that are multiples of $\frac{1}{2}$. Turning on a non-integer B_2-field is correlated with a choice of Chan-Paton bundle over the 2-cycle. To be precise, the bundle should be one without vector structure. The interpretation of the authors of [6] is that the correlation between B_2-field and Chan-Paton bundle is required by consistent coupling of closed strings to open strings. The anomaly of [17] can be interpreted in a similar way: trivializing the phase factor is necessary to couple the open strings ending on the D-brane to closed strings living in the bulk. If the factor cannot be trivialized, then the only option left is to remove the particular open string sector; in other words, to discard the possibility to wrap the brane around the 2-cycle. This truncation of the spectrum however may lead to other inconsistencies. For example, in the presence of orientifold planes D-branes are needed to ensure tadpole cancellation.

In this fashion the rank reduction in the case of toroidal compactification without vector structure can be understood as follows. In type I theory on a torus a half-integer B_2-field gives a phase factor that can be canceled in two ways: One can turn on a nonzero gauge field strength, or wrap the D9 branes twice around the corresponding cycle. It is clear that wrapping twice gives a solution that is lower in energy than the turning on of a field strength. However, locally a twice wrapped D-brane is indistinguishable from two once wrapped D-branes. In particular, we need only 16 D9-branes to cancel the $O9$ tadpole, instead of the usual 32.

Absence of vector structure for a $Spin(32)/\mathbb{Z}_2$ bundle is measured by a characteristic class, a $\mathbb{Z}_2$-valued generalized Stieffel-Whitney class $\tilde{w}_2$. As the choice of bundle is correlated with the choice of B_2-field, it follows that one may identify the B_2-field with $\tilde{w}_2$. It is an interesting observation that discrete moduli for string theory are correlated with topological invariants of particular gauge bundles. There also appear to be suggestive links between the 3-form in M theory, and the Chern-Simons 3-form associated to particular bundles [18,14].

In this paper we wish to describe type I on a 4-torus with a RR 4-form field turned on. As we will see, a nontrivial RR 4-form field indeed results in reduction of the rank of the gauge group. This is however not due to multiply winding branes. Why one nevertheless gets reduction of the rank will be explained in the next section.

Another interesting question is whether the discrete 4-form of type I theory can be identified with some topological invariant of the $Spin(32)/\mathbb{Z}_2$ bundle. A piece of evidence is the existence of flat nontrivial bundles that are intrinsically 4-dimensional [11]. In Appendix D of this paper "nontrivial quadruples" were constructed, 4-tuples of group elements such that every subset of these 4 elements can be chosen on a maximal torus of the group, but not all four simultaneously. Such a 4-tuple can be used for a compactification on the 4-torus, by choosing the elements of the 4-tuple as holonomies.

Bundles over the 4-torus parametrized by such a 4-tuple are intrinsically 4-dimensional (as the bundle over every sub 3-torus is a standard compactification). One may speculate on the existence of a 4-dimensional topological invariant, distinguishing the compactification with a nontrivial quadruple form a trivial one. Unfortunately, such an invariant has not (yet) been constructed. This is to be contrasted with compactifications of gauge theories on lower dimensional tori, where the invariants classifying the bundles are understood, and do allow generalization to nontoroidal compactifications. We will argue that type I string theory has a candidate for such an invariant, provided by the RR 4-form. We will show that turning on this form reproduces the nontrivial quadruple of [11]. It is also possible to combine nonzero expectation values for both the B_2-field and the RR 4-form, leading to a "nontrivial quadruple without vector structure" as first encountered in [14].

We note that the fact that we are not going to find multiply wrapped branes can also be heuristically understood from the link to bundles in gauge theories. Nontrivial quadruples in the sense of [11] only exist in (large enough) orthogonal groups. When extending to "c-quadruples" (which we define extrapolating on the definitions of [12], as 4-tuples of elements which commute up to elements of the center of the simply connected cover of the group), it is possible to show that these may also appear in theories with symplectic groups. Important however is, that it is simple to show that they do not occur for the unitary groups. Translated to string theory this suggests that the orientifold projection is essential to the effects of the RR 4-form, at least in the context that we wish to study. Of course this makes the effect of the RR 4-form in absence of an orientifold projection even less understood.

III. IDENTIFICATION VIA *T* DUALITY

Consider type I on a 4-torus, with possibly some B_2-fields, and possibly the RR 4-form turned on. There are, up to $SL(4,\mathbb{Z})$ transformations, three possibilities for B_2-flux over the 4-torus, that can be distinguished as follows. Viewing B_2 as a 2-form with integer periods, one has the possibilities

$$B_2=0; \quad B_2\neq 0, \quad B_2^2=0; \quad B_2\neq 0, \quad B_2^2\neq 0. \qquad (3)$$

These are of course in precise correspondence with the topological choices for the $Spin(32)/\mathbb{Z}_2$ bundle over the 4-torus [7].

On the 4-torus, a 4-form is invariant under $SL(4,\mathbb{Z})$ transformations (because it has to be proportional to the volume form). Calling the 4-form C_4, there are 2 possibilities:

$$C_4=0; \quad C_4\neq 0. \qquad (4)$$

Hence *a priori* there are 6 possibilities to consider.

Applying 4 *T* dualities to the type I theory on the 4-torus gives us a IIB orientifold on $T^4/\mathbb{Z}_2$ with 16 $O5$ planes at the fixed points of the $\mathbb{Z}_2$ action. The identities of the $O5$ planes depend on the fluxes in the parent type I model. The 4 *T* dualities result in a B_2-field background that is identical to that of the parent model on T^4.

There are 2 possible discrete charges for $O5$ planes: its transverse space is $\mathbb{RP}^3$, which has the following interesting cohomologies. There is a possibility for a discrete charge for the NS-NS 2-form, as its class

$$[dB_2]=[H_3]\in \tilde{H}^3(\mathbb{RP}^3)=\mathbb{Z}_2. \qquad (5)$$

Another possibility is a discrete charge for the RR scalar, as

$$[dC_0]=[G_1]\in \tilde{H}^1(\mathbb{RP}^3)=\mathbb{Z}_2. \qquad (6)$$

By these two $\mathbb{Z}_2$ charges, it is possible to distinguish 4 types of O5-planes which we will denote as $O5^-$, $O5^+$, $\widetilde{O5^-}$ and $\widetilde{O5^+}$. Here we follow the notation of [19], using the superscript + for planes with nontrivial NS-charge, and a tilde for planes with nontrivial RR charge. The D5 brane charges of these orientifold planes are -2 for the $O5^-$, -1 for the $\widetilde{O5^-}$, and $+2$ for the $O5^+$, $\widetilde{O5^+}$. The difference in charge between $O5^-$ and $\widetilde{O5^-}$ gives rise to the interpretation of the second as a bound state of an $O5^-$ with a single D5-brane. The D5-brane charge of $O5^+$ and $\widetilde{O5^+}$ is the same, and the two can presumably only be distinguished by nonperturbative effects. Nevertheless, we will find that both make a natural appearance.

The distribution of the NS-charges over the $O5$ planes in the IIB orientifold follows from the B_2-fluxes in the original type I model [7]. The 3 cases give

B_2-flux in type I on T^4	$B_2=0$	$B_2\neq 0,\ B_2^2\neq 0$	$B_2\neq 0,\ B_2^2\neq 0$
planes with nontrivial B-charge	0	4	6

In the case of 4 planes with nontrivial NS-charge, the intersection points of these 4 $O5$-planes are aligned within a 2-plane. In the case of 6 planes with nontrivial charges, the 6 planes are aligned in two 2-planes intersecting in a point, with a plane with trivial NS-charge at the intersection point.

The RR 4-form of the type I theory on the 4-torus results after 4 T dualities in a constant RR scalar background for the IIB orientifold on $T^4/\mathbb{Z}_2$. This inevitably implies that all orientifold planes have the same RR-scalar charge, as it can be measured at any point near the $O5$-plane.

In a recent paper [20] also the possibility of gradients for the RR-scalar between $O5$-planes was considered. Such a gradient gives a nonzero field strength, and via coupling to gravity should modify the curvature of space-time. Solutions to the combined problem (solving the Einstein equations, for a space with some compact directions, and with appropriate symmetries such that the orientifold planes can be inserted) would be very interesting, but probably break some supersymmetry, and it seems unlikely that they are related to the type I string in a simple way. We will therefore discard this possibility.

The 6 possibilities for the duals to the type I string compactification on a 4-torus have the following configurations of $O5$-planes:

All 16 planes are $O5^-$ planes. This is the trivial compactification, and there are 32 D5-branes, arranged in 16 pairs, giving a rank 16 gauge group.

4 $O5^+$ planes and 12 $O5^-$ planes. This is dual to the compactification without vector structure discussed in [7]. There are 8 pairs of D5-branes.

6 $O5^+$ planes and 10 $O5^-$ planes. This is another compactification that was briefly described in [7]. There are 4 pairs of D5-branes.

All 16 planes are $\widetilde{O5^-}$ planes. This compactification is dual to a type I compactification with a nontrivial quadruple. It is trivial to reconstruct the holonomies for this case, and compare them with [11]. Note that there are 32 D5-branes, of which 16 form bound states with the $O5$ planes, while 16 others are arranged in pairs. Therefore the rank of the gauge group is 8, and, remarkably, it can actually be demonstrated that this orientifold is in the same moduli space as the one with 4 $O5^+$ planes and 12 $O5^-$ planes [14].

4 $\widetilde{O5^+}$ planes and 12 $\widetilde{O5^-}$ planes. This model only appeared previously briefly in [14]. It has 16 D-branes like the standard compactification without vector structure, of which 12 are bound to $O5$ planes, and 4 are arranged in pairs. Therefore the rank of the gauge group is only 2. In [14] it was conjectured to be dual to the type I compactification with a quadruple without vector structure. In the next section we will give compelling evidence for this duality by reconstructing the Wilson lines from this orientifold.

The last model would have 6 $\widetilde{O5^+}$ planes and 10 $\widetilde{O5^-}$ planes. Now the tadpole can only be canceled by adding anti D5-branes in pairs, which should then annihilate with the single D-branes stuck to the $O5$ plane, resulting in a model where some of the $O5$ planes form bound states with an anti D5-brane. Actually, if this is a valid possibility, then it is impossible to tell which $O5$ planes have the anti D5-branes, because all possibilities can be realized. In fact, they should be realized, because by D5 brane-antibrane pair creation in the bulk, and letting these annihilate with branes and antibranes bound to $O5$-planes, the anti-D5-brane can ''jump'' to other $O5$-planes. The true ground state of the theory would then be a superposition of all possible configurations.

Note that we ''derived'' this non-supersymmetric model from a type I compactification with certain background fields. It may come as a surprise that apparently one can break supersymmetry by turning on background B_2 and C_4 fields, that carry no field strength. This is an important lesson.

Incidentally, we note that a model with 6 $\widetilde{O5^+}$ planes and 10 $\widetilde{O5^-}$ planes describes a perfectly sensible gauge bundle for $Spin(N\geqslant 40)$ gauge theory (compare with [16]). It is just the fact that the (perturbative) gauge group of type I is ''not big enough'' that leads to the subtleties mentioned.

For completeness we mention that there exist two more type IIB orientifolds on $T^4/\mathbb{Z}_2$ (with 16 supersymmetries), both with 8 $O5^+$ planes and 8 $O5^-$ planes. It was noted in [14,21] that there already exist two geometrically inequivalent configurations for type IIA on $T^3/\mathbb{Z}_2$ with 4 $O6^+$ and 4 $O6^-$ planes. Compactifying these on a circle and T dualizing leads to two inequivalent configurations on $T^4/\mathbb{Z}_2$. It is clear however that for these cases we cannot add an RR-scalar background, as this would lead to tadpoles or supersymmetry breaking.

The existence of inequivalent theories with an equal number of Op^+ and Op^- planes allows an elegant explanation. In [7] it is argued that these theories have their origin in a special orientifold of type IIB theory on a circle. Instead of orientifolding this theory straight away, the orientifold action Ω is combined with half a translation δ over the circle. Another, equivalent way of stating this is that it is type IIB on a circle with a special holonomy around the circle [16]. Just like in the case compactifications of the ''ordinary'' orientifold, which leads to type I theory, compactifications of this ''special'' orientifold of type IIB theory allow a nontrivial background B_2 field, but it can only take two discrete values. Compactifying type IIB on a circle, modded by $\delta\Omega$ on a further 2-torus, one has the choice of turning on a discrete B_2 field. Therefore in 7D, there are 2 inequivalent theories, which after dualizing result in the 2 inequivalent type IIA orientifolds with 4 $O6^+$ planes and 4 $O6^-$ planes. In the same fashion, one immediately sees that the type IIA orientifold on $T^5/\mathbb{Z}_2$ with 16 $O4^+$ and 16 $O4^-$ allows 3 inequivalent geometries [from the 3 inequivalent choices of B_2-field, as in Eq. (3)], the type IIA orientifold on $T^7/\mathbb{Z}_2$ with 64 $O2^+$ and 64 $O2^-$ comes with 4 inequivalent geometries, and type IIA on $T^9/\mathbb{Z}_2$ with 256 $O0^+$ and 256 $O0^-$ exists in 5 inequivalent geometries (more about this in [22]). We also note that these considerations make a nonperturbative equivalence of the different configurations with equalnumbers of Op^+ and Op^-, mentioned in [14] but conjectured to be false, indeed highly unlikely.

In total, we have identified 7 orientifolds on $T^4/\mathbb{Z}_2$ preserving 16 supersymmetries (it can actually be shown that these are all maximally supersymmetric orientifolds with 16 O5-planes only [22]).

IV. THE MODEL WITH $\widetilde{O5^+}$ PLANES

In this section we will take a closer look at the model on $T^4/\mathbb{Z}_2$ with 4 $\widetilde{O5^+}$ planes and 12 $\widetilde{O5^-}$ planes. This appears to be the only supersymmetric model in which $\widetilde{O5^+}$ planes quite naturally appear. $\widetilde{Op^+}$ planes with $p<5$ have been studied before [19,23,24].

Our explanation for the appearance of the $\widetilde{O5^+}$ originates in the B_2 and C_4 holonomies appearing in the type I model. The NS-charge of the $\widetilde{O5^+}$ can also be confirmed by studying the low energy gauge theory, as it is supposed to lead to $Sp(n)$ gauge symmetry. The RR-charge cannot be confirmed this way, as both $\widetilde{O5^+}$ and $O5^+$ lead to the same low energy gauge group, $Sp(n)$. But there is another piece of evidence that (we think) supports our assignment of charges.

In [14] the 2 models with 4 $O5^+$ planes and 12 $O5^-$ planes, and 16 $\widetilde{O5^-}$ planes, where shown to be in the same moduli space. The most unambiguous way to show this is to translate both models to heterotic string theories, whose equivalence can be demonstrated exactly [14]. Another, less precise way to exhibit the close relationship between the two models is to compactify both on an additional 2-torus, and T dualize along the 2 directions of this torus. This leads to type IIB orientifolds on $T^6/\mathbb{Z}_2$ with 16 $O3^+$ planes and 48 $O5^-$ planes, and another with 16 $\widetilde{O3^-}$ planes and 48 $O3^-$ planes. These are dual to each other by S-duality of 4D $N=4$ supersymmetric gauge theories, which is realized as a $\mathbb{Z}_2$ involution on the component of the string moduli space that contains the CHL-string, as well as the above two models [25,26,14].

It is interesting to apply the same procedure to the model with 4 $\widetilde{O5^+}$ planes and 12 $\widetilde{O5^-}$ planes. Compactification on a 2-torus, and applying T dualities twice results in a model on $T^6/\mathbb{Z}_2$ with 4 $\widetilde{O5^+}$ planes, 12 $\widetilde{O5^-}$ planes, 12 $O5^+$ planes and 36 $O5^-$ planes. This model is self-dual under S duality of 4D $N=4$ supersymmetric gauge theories. Indeed, in [14] this model was conjectured to be dual to the $\mathbb{Z}_4$ triple construction in the $E_8\times E_8$ heterotic string. The component of the string moduli space that contains these theories is mapped to itself under the $\mathbb{Z}_2$ involution implied by S duality of 4D $N=4$ theories. The self-duality under 4D S duality of the compactification to 4D of the orientifold with 4 $\widetilde{O5^+}$ planes and 12 $\widetilde{O5^-}$ planes is clearly consistent with the other proposed dualities.

Although in principle the RR-charges of the Op^+ planes could also be determined by studying the monopole spectrum in $d=3$ [19], this is very subtle. The reason for this is that there are only 2 D-brane pairs present in the theory, and hence the only gauge groups one can get at Op^+ planes are $Sp(1)$ and $Sp(2)$. Therefore the groups of the monopole theory can only be $SO(5)=Sp(2)/\mathbb{Z}_2$ and $SO(3)=Sp(1)/\mathbb{Z}_2$. Hence, a determination of the gauge groups appearing in the S dual theories is not enough, one really needs to study the topology of the gauge groups in detail. Together with the fact that in type I theory and its duals the topology of the gauge group is different from the one that is manifest in perturbation theory [2] (note also the issues raised on the topology of the gauge group in [14]), the analysis appears to be very difficult, and not necessarily decisive. We will not attempt such an analysis here.

We will now reconstruct the same model in another way, which clarifies the duality to the $\mathbb{Z}_4$ triple in $E_8\times E_8$ heterotic string theory.

We start again with type I string theory on a 4-torus. Start by turning on a Wilson line on the first circle which breaks the gauge group to a group that can be shown to be $[Spin(16)\times Spin(16)]/\mathbb{Z}_2$ (see [14] for a discussion on the topology of subgroups in string theory). This group is the one that is often denoted as "$SO(16)\times SO(16)$," for example in discussions on T duality of heterotic theories. T dualizing along this direction now leads to a model where one half of the D-branes is localized at $x_1=0$ and the other half is at $x_1=\pi$ (in an obvious choice of coordinates).

The point is that $Spin(16)/\mathbb{Z}_2$ allows various triples without vector structure [12]. There are 4 distinct possibilities. An example of the first kind combines 2 holonomies parametrizing absence of vector structure, while taking for the third holonomy the identity. By continuous deformations one can reach other models on the same component of the moduli space. The Chern-Simons invariant for these triples is integer [12]. This is the only triple without vector structure that is allowed in string theory [14]. The other ones are nevertheless useful as building blocks, as we will demonstrate.

A second example combines 2 holonomies parametrizing absence of vector structure, with a third holonomy corresponding to the nontrivial element of the center of $Spin(16)/\mathbb{Z}_2$. Again the rest of this component in the moduli space can be covered by continuous deformations. This component has Chern-Simons invariant equal to $\frac{1}{2}$ plus some integer, and we will not discuss it further.

The third and fourth example are more interesting. By choosing an appropriate Wilson line, it is possible to break $Spin(16)/\mathbb{Z}_2$ to $[SU(4)\times Spin(10)]/\mathbb{Z}_4$ (with the $\mathbb{Z}_4$ acting diagonally on both factors). Choosing this element as our third holonomy, and two other holonomies that commute up to an element of order 4 in the $\mathbb{Z}_4$ that was divided out of $SU(4)\times Spin(10)$, one finds 3 holonomies that commute in $Spin(16)/\mathbb{Z}_2$. They do not commute when lifted to $Spin(16)$ and therefore define a triple without vector structure. There are basically two options (because there are 2 elements of order 4 in $\mathbb{Z}_4$) leading to Chern-Simons invariants of $\frac{1}{4}$ and $\frac{3}{4}$ up to integers. These models do allow an orientifold description, which was constructed in [16] (section 4.3.2).

Compactifications with noninteger Chern-Simons invariants are not allowed in consistent string theories [14]. Here however, we have the group $[Spin(16)\times Spin(16)]/\mathbb{Z}_2$, which allows us to embed a triple without vector structure in each $Spin(16)$ factor. Choosing the triple with Chern-Simons invariant $\frac{1}{4}$ in one factor, and the triple with Chern-Simons $\frac{3}{4}$ in the other, all possible Chern-Simons invariants that can be defined over the 4-torus are integer, and these holonomies define a consistent string background. This also illustrates the equivalence of the present construction to the formulation of the $E_8\times E_8$ $\mathbb{Z}_4$-triple theory, that appears in [14]. Finally, using the (inconsistent) orientifolds parametrizing the triple theories that were constructed in [16], one can trivially construct the (consistent) orientifold representation of the present quadruple without vector structure: We had half of our D-branes living at $x_1=0$ and one-half at $x_1=\pi$, and copying the orientifold description for the triple theories leads immediately to an orientifold on $T^4/\mathbb{Z}_2$ with 12 $\widetilde{O5}^-$ planes. Furthermore there are 4 planes that are either $\widetilde{O5}^+$ or $O5^+$ planes. This cannot be decided from an analysis of the gauge group, but we have given other evidence that these actually should be $\widetilde{O5}^+$ for a consistent string theory.

To complete this section, we will deduce the holonomies parametrized by the type IIB orientifold on $T^4/\mathbb{Z}_2$ with 4 $\widetilde{O5}^+$ and 12 $\widetilde{O5}^-$ planes. One can use the techniques described in [16]. We start by first defining some building blocks

$$A=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad B=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix},\quad C=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \tag{7}$$

$$D(\phi)=\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}. \tag{8}$$

The holonomies can then be expressed in a relatively compact way as

$$\Omega_1=(A\oplus B\oplus C)^4\oplus(D(\phi_1)\otimes A)\oplus(D(\psi_1)\otimes A) \tag{9}$$

$$\Omega_2=(B\oplus A\oplus A)^4\oplus(D(\phi_2)\otimes C)\oplus(D(\psi_2)\otimes C) \tag{10}$$

$$\Omega_3=\mathrm{diag}(1^{12},(-1)^{12})\oplus(D(\phi_3)^2)\oplus(D(\psi_3)^2) \tag{11}$$

$$\Omega_4=\mathrm{diag}(1^6,(-1)^6,1^6,(-1)^6)\oplus(D(\phi_4)^2)\oplus(D(\psi_4)^2). \tag{12}$$

The notation with superscripts indicates that the corresponding arguments have to be repeated. The arguments ϕ_i and ψ_i are the coordinates of the 2 pairs of D-branes on the orientifold, suitably normalized. The reader may verify that Ω_1 and Ω_2 anticommute, that all other combinations of holonomies commute, and that the eigenvalues of the holonomies coincide with the ones given for the heterotic $Spin(32)/\mathbb{Z}_2$ with a quadruple without vector structure as given in [14], translated to the vector representation of $Spin(32)$.

V. OTHER MODULI ?

At least naively, the reasoning that leads to considering the possibility for non-trivial discrete NS 2-form and RR 4-form backgrounds in the type I string seems to suggest still more possibilities for discrete moduli. One may also study type I theory on a sufficiently large torus, with nontrivial backgrounds for the RR 8-form and the NS 6-form. After applying T dualities, this presumably leads to the additional variants for orientifold lines and points considered in [19] (compactifications with nontrivial NS 6-form background are currently under study [27]).

In view of the above it is natural to ask whether one can turn on a background for the RR-scalar. After T dualities this would lead to RR p-form charges for $O(9-p)$ planes. These charges do not appear in [19]. This is because such fluxes would fill the whole transverse space to the orientifold plane, and in particular can not be studied with the cohomologies of the $\mathbb{RP}^{8-p}$ that surrounds the Op-plane. This is not necessarily an obstruction to their existence, as the same problem exists for $O8^+$ and $O7^+$ planes, and can be overcome (see e.g. [7,14]). We note that this may imply that Op planes with B_6 fluxes may exist in dimensions higher than 1.

At least in some cases it seems to be possible to make sense out of orientifold Op planes with C_{9-p}-flux (where C_{9-p} denotes a transverse RR $9-p$-form flux). The first example that comes to mind is by using type IIB S duality on a $O7^+$ plane. In this case one can easily determine the charges, the tension and the gauge group associated to the resulting plane.

As a second example we consider C_1-flux for $O8$-planes. There exist variants of $O4$ planes and $O0$-points that carry a similar flux. When lifting these planes to M theory, C_1 becomes a component of the metric. In particular, it can be demonstrated that the $O4$ and $O0$ lift to M theory on [23,24,19]

$$\mathbb{R}^{4,1}\times(\mathbb{R}^5\times S^1)/\mathbb{Z}_2 \quad \text{and} \quad \mathbb{R}^{0,1}\times(\mathbb{R}^9\times S^1)/\mathbb{Z}_2.$$

Here the $\mathbb{Z}_2$ acts as a shift on S^1 and as a reflection on R^n. In full analogy, an $O8$ with nontrivial C_1 charge should lift to M theory on

$$\mathbb{R}^{8,1}\times(\mathbb{R}^1\times S^1)/\mathbb{Z}_2.$$

But in that case, this object is already known, as this description applies (locally) to M theory on a Klein bottle [28], and on a Möbius strip. Such an $O8$-plane would carry D8 brane charge 0.

A third (more speculative) example is provided by C_3-flux for $O6$ planes. In [14], M theory on a K3 with background 3-form fluxes was considered. For a background $\mathbb{Z}_2$ valued 3-form flux, one needs an even number of frozen D_4 singularities. If one could split of a circular fiber, in such a way that pairs of D_4 singularities are located in the same fiber, then presumably the theory can be reduced to a type IIA theory, with a 3-form flux background. Comparing various charges, the fiber with two D_4 singularities reduces to an object with D6 brane charge 12, and a nontrivial 3-form charge. Lifting a single object in type IIA theory to two singularities may seem unusual, but the reader may wish to compare with the case of the $O6^-$ with 4 D6-branes on top, that lifts to two A_1 singularities.

There seems to be a pattern consisting of Op-planes, with Dp brane charge $16-2^{p-4}$, and RR $(9-p)$-form charge. Is it possible to have an $O9$ plane with these charges? For several reasons, this is problematic. First of all, such an $O9$-plane could be used to construct a new 10 dimensional open string theory with a rank 8 gauge group, but this theory is not known. Second, this theory would appear (via M theory on the Möbius strip) to be a 10 dimensional limit of the CHL-string, but various arguments (see e.g. [26]) indicate that such a limit does not exist. This suggests that the discrete C_0 flux can only be defined on a manifold with compact directions. It would be interesting to investigate this possibility further.

A serious drawback of the previous considerations is that we more or less ''define'' variants of orientifold planes by projections from nonperturbative descriptions [type IIB $SL(2,\mathbb{Z})$ duality, M theory]. This is opposite to common practice, where the perturbative objects are well defined, and one tries to deduce the description in the strong-coupling regime. To some extent, the question is whether these variants of orientifold planes (and the ones introduced in [19]) are really sensible as string theory objects, or whether they only start to make sense in a more complete, nonperturbative description of string theory.

VI. DISCUSSION AND CONCLUSIONS

We have demonstrated that type I compactifications on a 4-torus with a nontrivial RR 4-form background field lead to theories with gauge groups of reduced rank. The RR 4-form was shown to be correlated with compactifications with ''nontrivial quadruples'' of holonomies. It is not known at present whether there exists a characterization of these bundles that extends to other 4 manifolds. For example, it would be very interesting to compactify type I theories on K3, or a Calabi-Yau manifold with 4-cycles, and turn on the discrete 4-form background, but at present we have no clue how to describe such compactifications.

In one of the T-dual models, the $\widetilde{O5^+}$ makes a natural appearance. The existence of this plane was deduced before, from an analysis of possible discrete charges, but it has not appeared thus far in an explicit model. This particular model on $T^4/\mathbb{Z}_2$, with 4 $\widetilde{O5^+}$ and 12 $\widetilde{O5^-}$ planes is related by dualities to a $\mathbb{Z}_4$ asymmetric orbifold of the heterotic string [14]. There is much known about compactifications of the heterotic string with maximal supersymmetry but gauge groups with reduced rank gauge [29,25,30]. It is even possible to reduce the rank to zero, eliminating even the Kaluza-Klein bosons, such that the resulting low-energy $N=4$ theory contains no vector multiplets at all [29]. Heterotic theories with reduced rank can be constructed by fixed point free asymmetric orbifolds of the heterotic string, and many group actions are possible [30]. In contrast, only those constructions that result from group actions that are products of $\mathbb{Z}_2$ factors had been explored for the dual type I string [7]. In this paper, and in [14] it was demonstrated that also the $\mathbb{Z}_4$ asymmetric orbifold of [30] has a dual description as type I on a 4-torus with a special bundle. The reader may wonder whether also asymmetric orbifolds of the heterotic string with other groups can be realized as type I compactifications. This question is not easy to answer, as we will explain now.

The duality between the type I string and the heterotic string is a complicated one. The low energy effective theories match of course, but a number of features, such as the topology of the gauge group, are manifest in the perturbative heterotic description, while the type I description relies on nonperturbative input in the form of certain solitons and instantons [1,2]. In some sense, the situation gets worse if one compactifies. The heterotic description, due to Narain [31], treats Kaluza-Klein bosons and ''10-dimensional'' gauge bosons democratically. In fact, what one considers to be ''10-dimensional'' fields depends on ones point of view, compactification of the either the heterotic $E_8\times E_8$ or the $Spin(32)/\mathbb{Z}_2$ string can lead to the same theory [32]. The same is true for some asymmetric orbifolds of the heterotic string, which can be interpreted as either heterotic theory with a specific bundle [33,7,14]. On the contrary, the perturbative type I description (and dual type II orientifolds) keeps a set of 10-dimensional gauge fields manifest, part of the gauge symmetry is encoded in nonperturbative objects, and Kaluza-Klein gauge symmetries are not manifest at all. Yet, translating to the heterotic side, a generic asymmetric orbifold would mix all of these gauge bosons. It is clear that on the type I side this would lead to a very messy description, unless one can find a part of the moduli space where perturbative gauge fields, non-perturbative solitons and Kaluza-Klein bosons do not mix. The analysis of symmetries that can be realized on $Spin(32)$ bundles [10–12] (see also [14] for an overview) suggest that only asymmetric orbifolds that use $\mathbb{Z}_2$ and $\mathbb{Z}_4$ symmetries can be candidates for such ''simple'' descriptions. In such cases it should also be possible to extend the symmetries to the nonperturbative excitations of the theories, as sketched in the appendix of [34]. Note that we are not claiming that other asymmetric orbifolds cannot be realised as type I compactifications; such a claim would contradict heterotic-type I duality. Nevertheless, the tension between the manifest symmetries in the type I description, and the symmetry one needs for the orbifold severely complicates the construction.

In another type I model we found that supersymmetry was broken, even though we compactified on a torus with background fields without field strength. This is related to the subtle correlations between background fluxes and D-brane charges, and deserves further analysis. A K-theory analysis might shed some light here.

We briefly discussed some aspects of variants of orientifold planes, in particular Op-planes with C_{9-p} charge. The fact that it appears to be much easier to define these objects from M theory or using S dualities in type IIB theory, warrants the question whether they are really well defined in perturbative string theory. Instead it does not seem unlikely that only a nonperturbative description reveals all the properties of these planes.

ACKNOWLEDGMENTS

We would like to thank J. de Boer, R. Dijkgraaf, A. Hanany, D. Morrison, and S. Sethi for helpful conversations. This work is partly supported by EU-contract HPRN-CT-2000-00122.

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